

BALLOON TRACKING WITH MF RADAR

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Preface:

The "balloon" part was only discovered after the track was identified: so much for the "physical" meaning of the scatter motion; but the analysis and various small corrections and uncertainties are worth discussion for future reference. *CM.*

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I. Introduction:

This work started during a cursory glance at signal strength profiles (Figure 1), when the strange event on day 241 was noticed - apparently a scatterer moving down and then up in height - which is what the plot was designed to show. Subsequently, another format of plot which had been done previously was examined (Figure 2) which showed the same event. Then the Angle of arrival (AOA) and V_z (vertical/radial velocity) was calculated and showed that the event was due to a single moving scatterer - because plane phase fronts were found (from AOA phases), and that the motion didn't agree with the V_z variation. Since the phases on the main array (1.15 wavelength spacing) are not correct for zenith angles beyond ~ 30 deg, they were unfolded (by adding 0, +360, or -360 deg) in several ways, maintaining a plane phase front. One of the resulting tracks did agree with the V_z variation, and so this must be the correct one. Later it was found that a research balloon had been released that day, and this was it! So no physics unfortunately, but there is a chance to check the COHRTW phase data against a known scatterer.

II. Antenna system:

Figure 3 shows the transmitting and receiving arrays. The transmitting array consists of 16 pairs of orthogonal folded half-wave dipoles. The N-S and E-W sets are fed independently from the transmitter - usually just one set, in this case the N-S set, is used for transmitting. In the outer receiving "squares" parallel dipoles are hooked directly in parallel. Usually, and in this case, the N-S arrays are used for receiving. The height the transmitter and main receiving squares is approximately 75-80 ft. above ground except for the centre receiving dipole (50 ft.). This is close to $1/4$ wavelength (111 ft in free space) above the water table. The centre receiving antenna is a single dipole, only 50 ft. above ground - this leads to some complications in angle of arrival (AOA) determination later.

Figure 4 shows the combined pattern of the (N-S) transmitting and one outer receiver array - assumed to be co-located. These were placed $1/4$ wave height above a perfect ground, and the dipole radiation factor was included. The side-lobes north and east are ^{of} similar magnitude because the east lobe, expected to be larger due to the *lack of a* dipole factor, is reduced by cancellation in the spaced dipoles of the receiver antenna.

III. Phase calibration:

Phase calibration is done by forming histograms of antenna pair phase differences over a long period (e.g. months). The most probable phase difference is assumed to apply to the zenith. Since the signal path goes through different wide-band pre-amps in the receiver, depending on the gain control, histograms have been separated according to gain - although the difference is expected to be minor.

COHRTW SIGNAL(LIN.) 9-20 CSTI, ADJ. TO MAX (MIN=1 DOT)

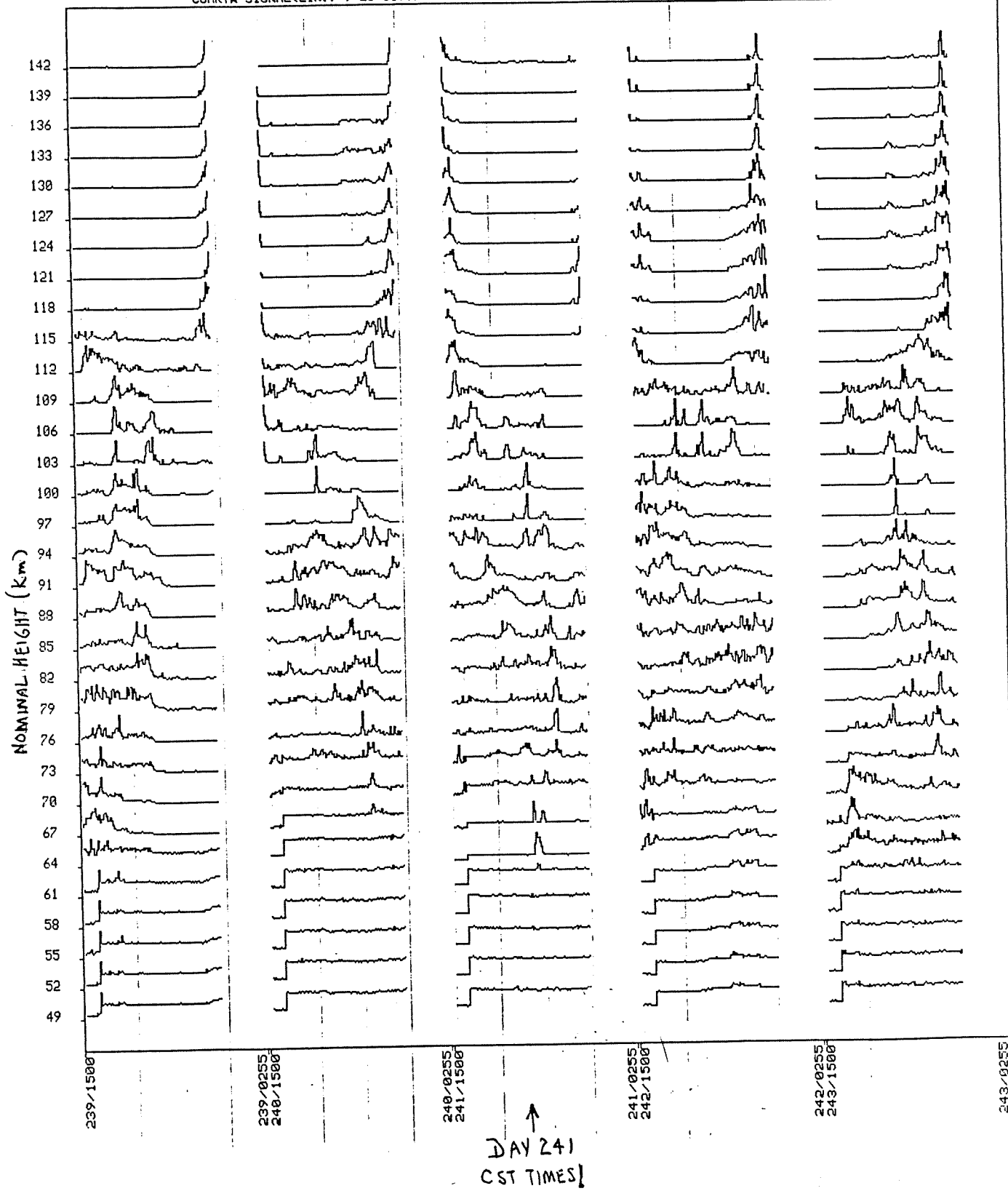


Figure 1.

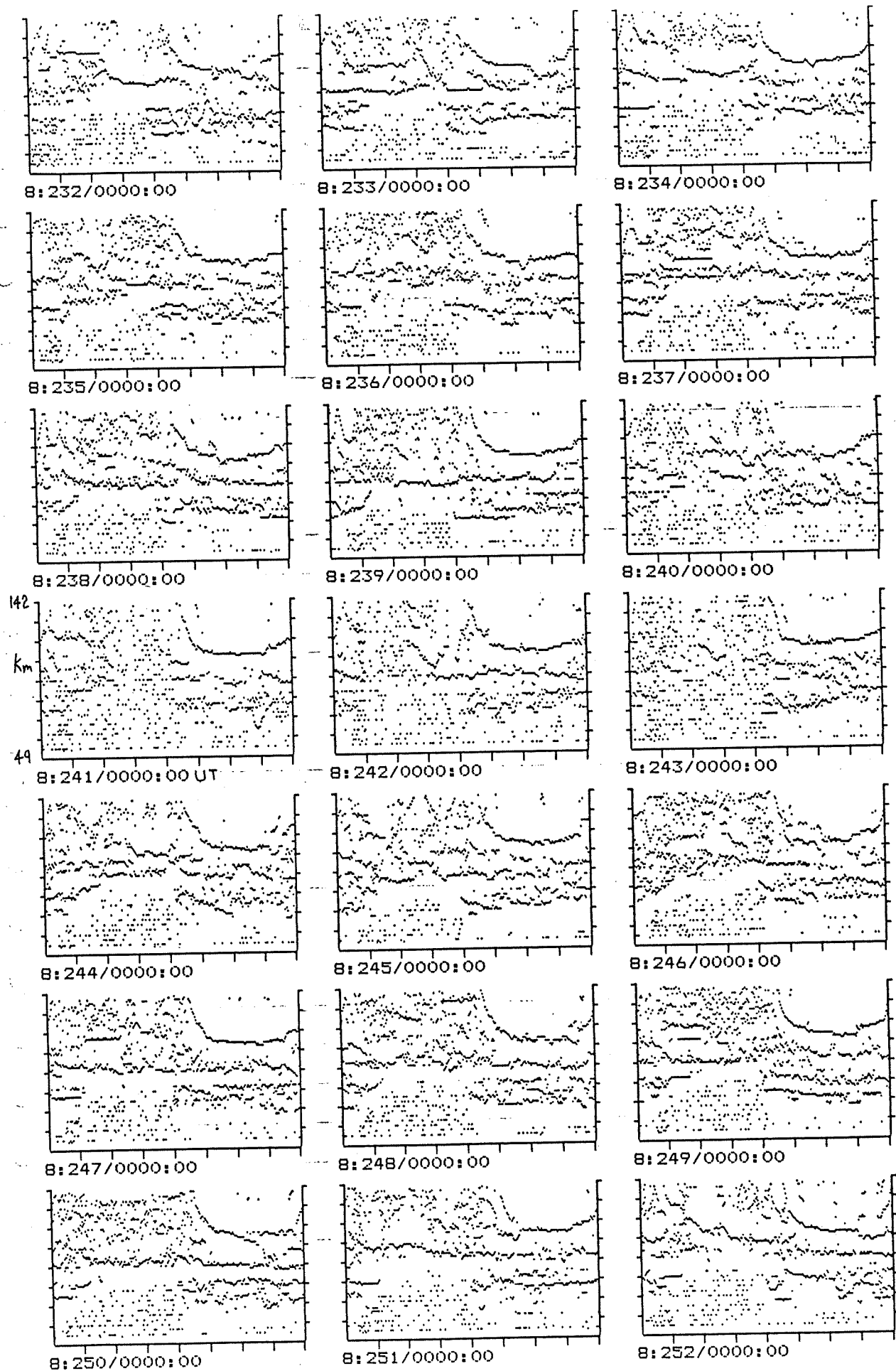


Figure 2.

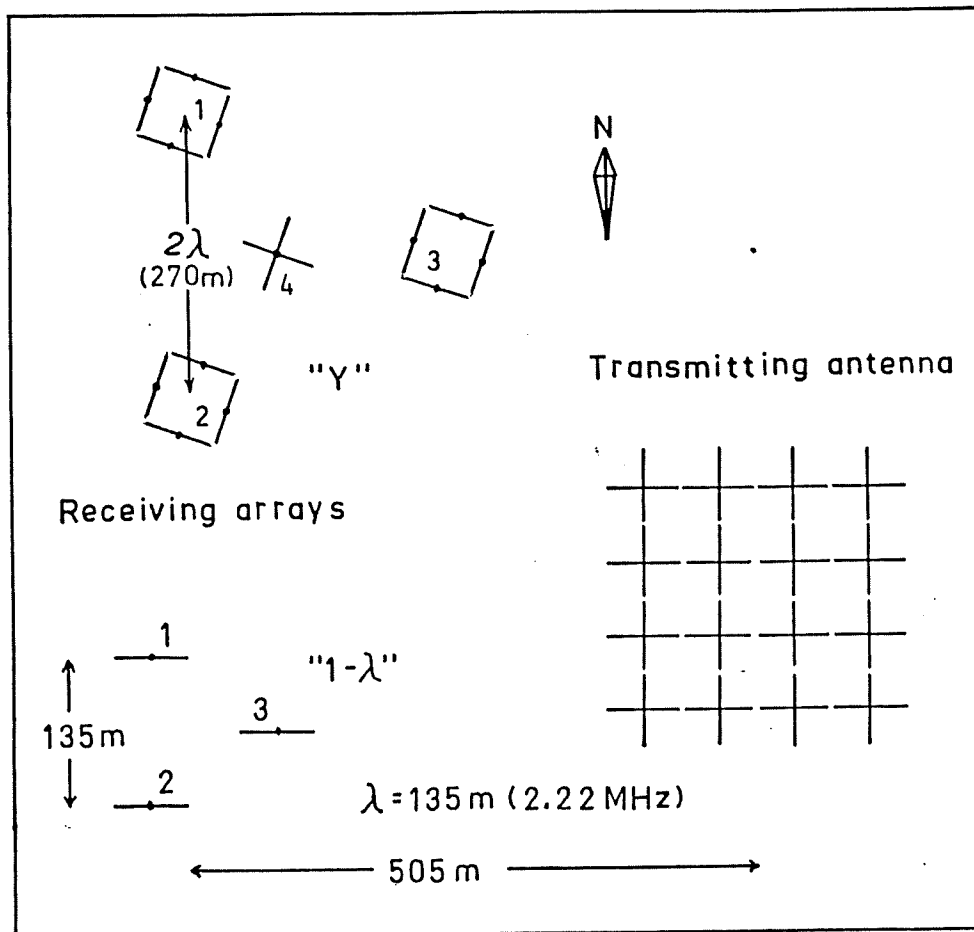


Figure 3.

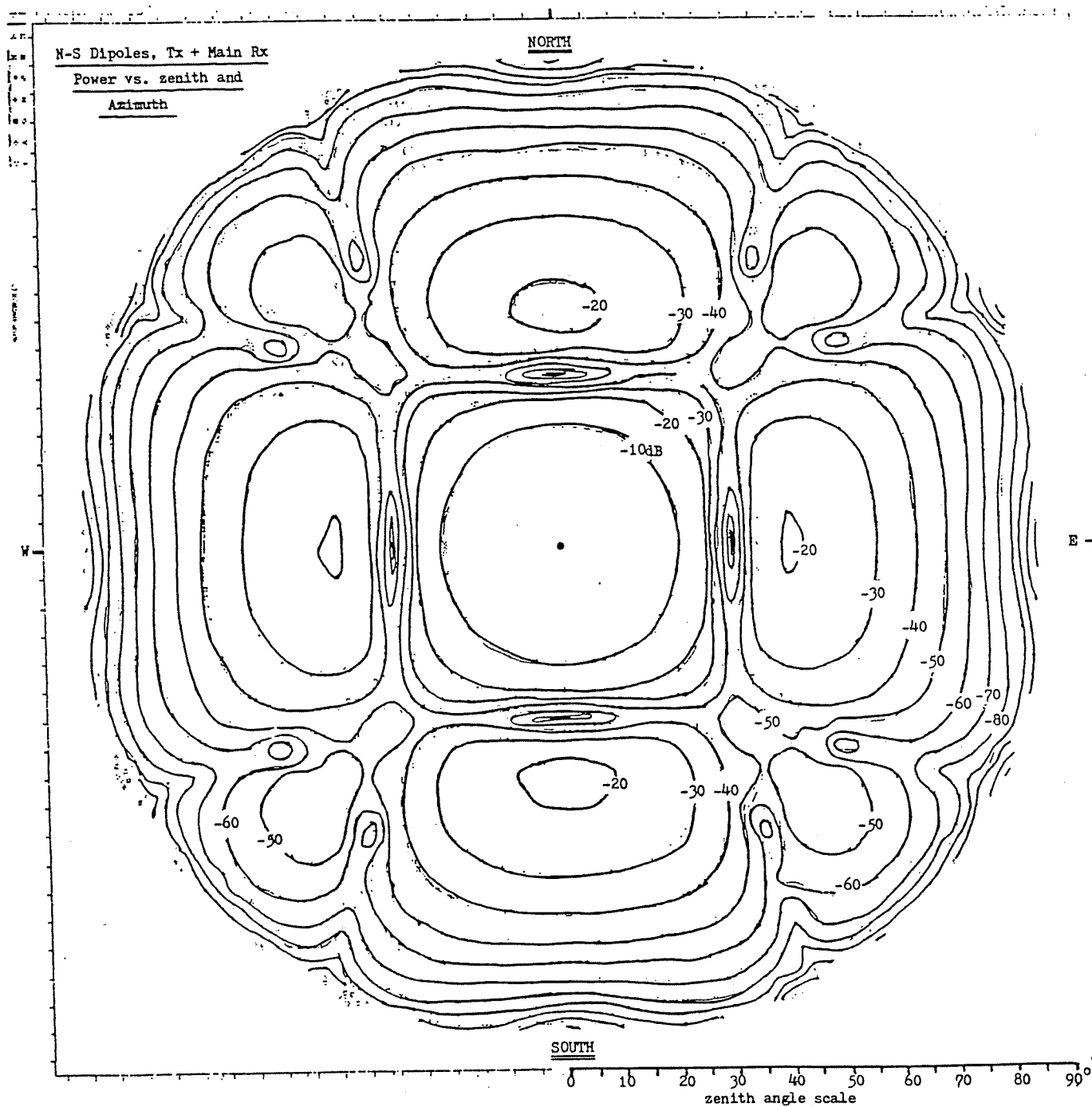


Figure 4.

Fig. 5,6 show some histograms for the higher gains. [Signals from the lower gains show a "tilt" - see Kyoto* paper; this might show a difference in scattering with height, since low gains are required at greater heights, or a difference in the receivers]. The most probable values for gain #7 are 314° , 302° , 294° . These are subtracted from the zero-lag phase measurements to get the "calibrated" phases used in AOA calculation. Table 1 shows the calibration phases for all gains: note that this should be checked for other specific intervals of interest - the calibration has changed at least once with no apparent reason.

Since the centre antenna (#4) on shorter poles than the rest, the calibration only applies accurately to signals from the zenith - corrections for non-zenith scatter will be discussed in the AOA calculation.

IV. AOA calculation

4.1 Direct Calculation

There are 3 antenna pairs available, but only 2 phase differences are needed to get the AOA, so a least squares fit is done to get the AOA which minimizes the squared error in phase differences. The case with all antennas at the same height is discussed first. Figure 7 shows the situation in spherical trigonometry. ψ_{4i} is the direction of the antenna pair vector (from antenna #4 to antenna #i), East of North.

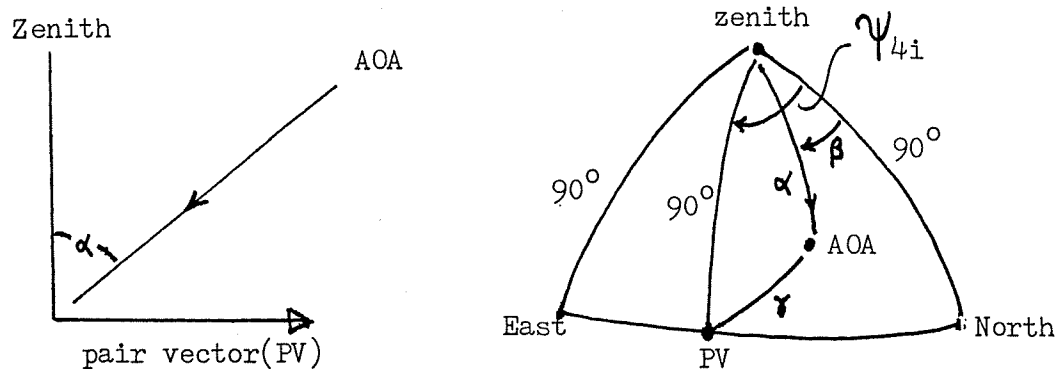


Figure 7.

The phase difference for this pair vector is

$$\Delta\phi_{4i} = -d \cos \gamma \quad (1)$$

The minus sign is to make the definition equal to the measuring convention : viz. if a phase difference increases positively, the AOA is leaning more towards antenna #4

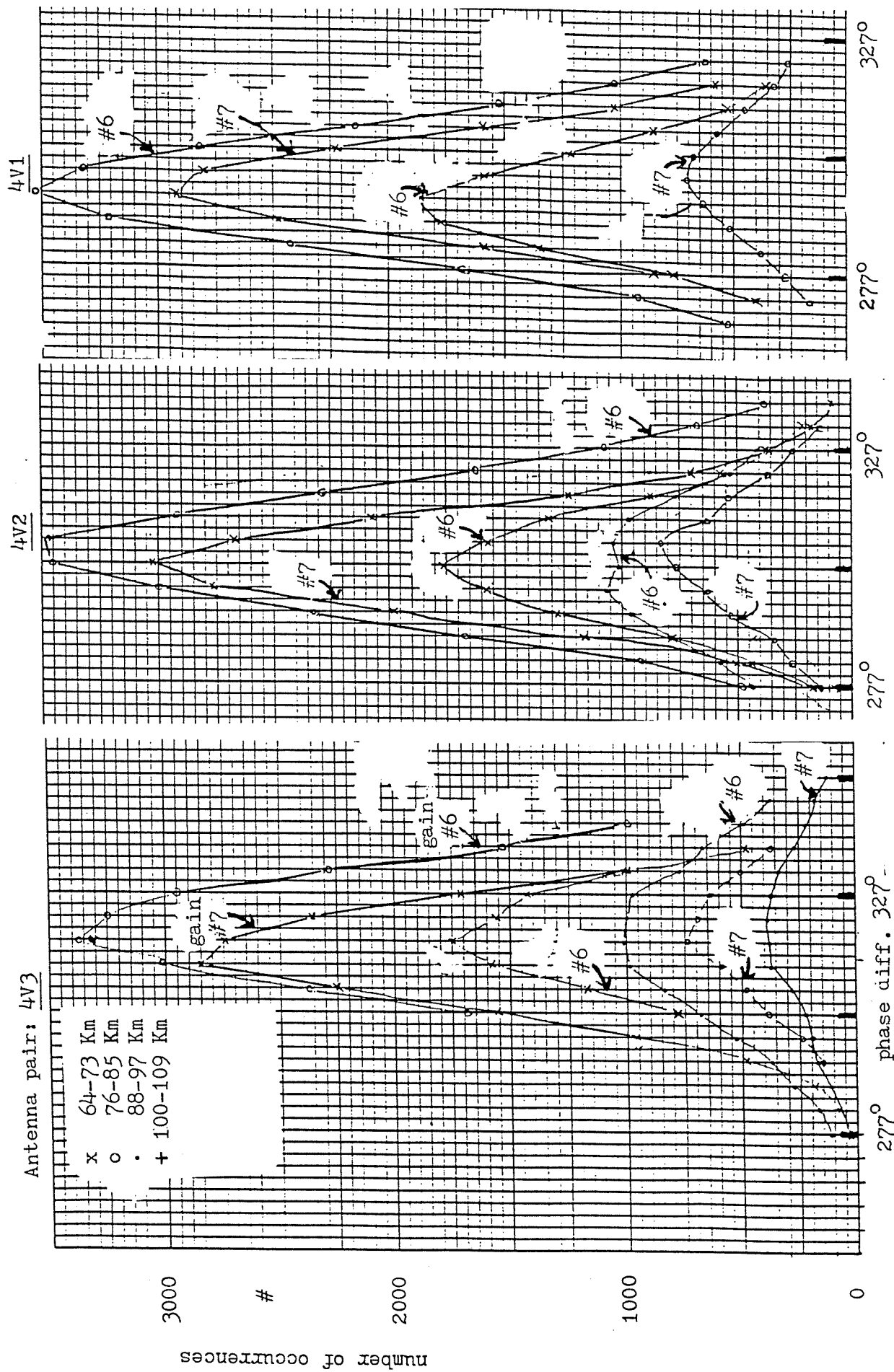


Figure 5,6. Phase calibration (just gains #6,7 shown) data: 1988:224-299, excl. 3-13UT, $p > 0.2$

Table 1. Phase (in degrees) calibration data (from normal COHRTW daytime data 88:224-299). "mp" indicates a skewed histogram and gives most probable value, # indicates a doubtful value. Gaps are left if not enough histogram points to get a good value. Correction is made in the sense: "data value minus calibration value". Note that these phase calibrations just apply to the stated period - they should be repeated for other periods of interest.

gain#	ant. pair	nominal height range (Km)			
		64-73	76-85	88-97	100-109

7	4V3	314			
6	"	317	319		
5	"		311	310	
4	"			307mp	
3	"				#302mp
2	"				304
1	"				294mp
0					

7	4V2	302	305		
6	"	302	304	303	
5	"		299	296mp	
4	"			294mp	287mp
3	"				282mp
2	"				279mp
1	"				279
0					

7	4V1	294	299		
6	"	290	293		
5	"	289mp	289	287	
4	"			284	#279mp
3	"				297mp
2	"				295mp
1	"				295
0					

From the cosine law in a spherical triangle

$$\cos \gamma = \cos 90^\circ \cos \alpha + \sin 90^\circ \sin \alpha \cos(\psi_{4i} - \beta) \quad (2)$$

where α is the zenith angle, β is the azimuth (E of N), and ψ_{4i} is the pair vector direction (from #4 to #i, E of N). This reduces to

$$\Delta\phi_{4i} = -d \sin \alpha \cos(\psi_{4i} - \beta) \quad (3)$$

which can be expanded to a linear equation

$$\Delta\phi_{4i} = -d \left[\cos \psi_{4i} [\sin \alpha \cos \beta] + \sin \psi_{4i} [\sin \alpha \sin \beta] \right] \quad (4)$$

A standard least squares fit finds the values in square brackets, from which the zenith and azimuth are easily obtained. A random set of phases can lead to an impossible AOA (i.e. $\sin \alpha > 1$.) and these must be rejected.

4.2 Phase folding:

The above assumes that the real phase differences are available. If the antenna pairs are spaced by more than 1/2 wavelength, this will not be the case for all AOAs, and the measured phases may be in error by $\pm 360n$, $n=0,1,2,\dots$. The number of possibilities is reduced by the necessity that the real phase difference must be equal or less than the antenna pair spacing (156m in this case, which is equivalent to 416° at 2.219 MHz), and also that the real phase differences must add to zero for scattering from a single AOA. An easy criterion to use for the latter is the NOD :

$$NOD = \frac{|\sum \Delta\phi_{4i}|}{\sum |\Delta\phi_{4i}|} \quad (5)$$

This has the advantage of always being between 0 and 1, but the disadvantage that its value decreases as the phases are unfolded (viz. the denominator always increases while the numerator may stay the same.

Figure 8 shows the NOD vs. zenith and azimuth for point scatter received by the "Y" array. In the central area, the phase differences are correct, and the AOA can be obtained immediately. Beyond this in the non-shaded areas, the NOD will still look good (i.e. single AOA), but the wrong AOA will be found. There is no way to distinguish between these two with just a single set of phase differences.

If a bad NOD is found with the initial (-180° to 180°) phases, then either it is a due to a single scatterer within the shaded area, or there is no single AOA.

Some other factors which may help to clarify the situation are the transmitter antenna beam width, the continuity of phases with time (suggesting a track), and if there is a track, the radial velocity may help to get the correct set of phases - as it did in the balloon case.

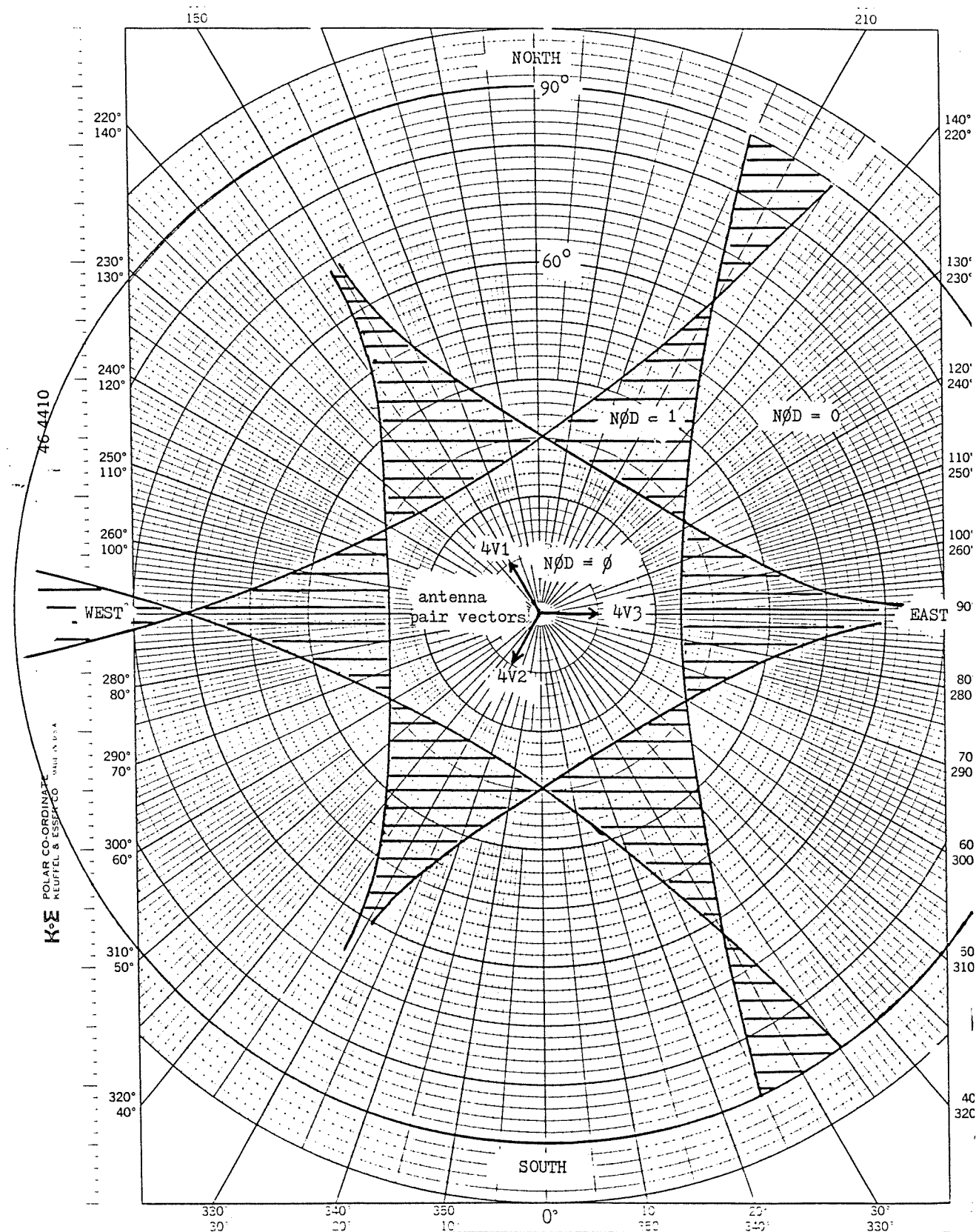


Figure 8.

4.3 AOA error due to centre antenna(#4)

The difference in height of the centre antenna relative to the outer ones is a major problem in analysis, although the correction to AOA is relatively small at low zenith angles (< 1 deg at 40° zenith). Figure 9 shows the spherical trigonometry for this case.

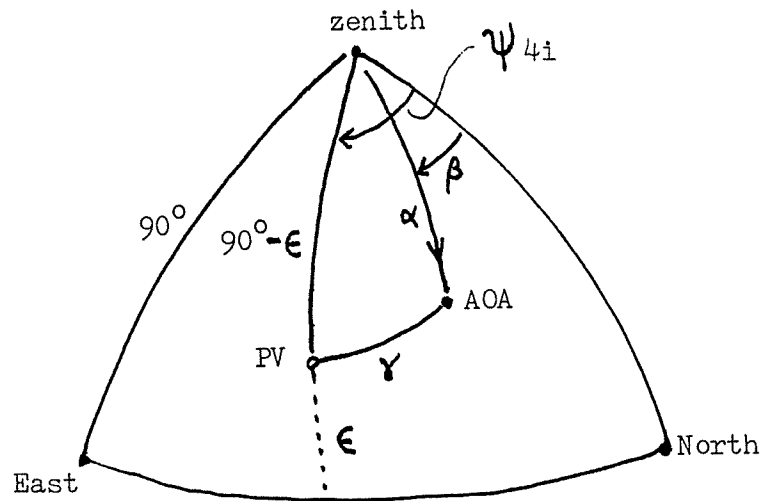


Figure 9.

ϵ is the elevation angle of a pair vector (centre antenna to outer). The phase difference is now:

$$\Delta\phi_{4i} = -d \left[\sin\epsilon \cos\alpha + \cos\epsilon \sin\alpha \cos(\psi_{4i} - \beta) \right] \quad (6)$$

This no longer resolves to a linear equation in two unknowns, and so a least squares fit is not simple. For the moment, a rough error estimate will be shown:

Figure 9 $\frac{1}{2}$ shows the situation of an AOA along an antenna pair. The calibration has added 20.4° (25 ft.) to the centre antenna path length, which factor is contained in the calibrated phase values.

This situation was set up so that for a real AOA, the "measured" phases were calculated, and then the AOA re-calculated from these assuming equal height antennas [The least squares fit had to be used since the NPD is not zero for these, except for scatter from the zenith]. Appendix A shows the results. The calculation shows that the phase error cancels completely! in the least squares fit (although it does increase the NPD) because the phase error is on the centre antenna - and so each phase difference is increased by the same amount.

this page reserved for fig 9 $\frac{1}{2}$

V. Range calculation:

The normal correction to nominal height is to subtract 3Km to get real height(range). This takes account of propagation delays in antenna feeders and receivers (but not necessarily the coherent receivers used here), and refers to the centre height of the scattering volume. In the balloon case, there is no scattering volume, and if the track is assumed to be a straight line, a more accurate value might be obtained by examining the part of the track at which the echo crosses from one height gate to the next - this range should be the nominal of the lower gate minus 1.5 Km.

VI. Data

Table 2 lists the COHRTW data for the balloon track - some of the extreme points (e.g. at 70,73 Km) were unfolded in such a way as to agree with the locations found at 64 Km. The table the original measured phases, and AOA parameters for the corrected & unfolded phases.

The 1/2 hour balloon positions received from Danny Ball (NASA, Palestine, Texas) are:

2030UT: 52 13'N, 106 28'W
2100UT: 52 25'N, 106 29'W

Park is located at 52 12.4'N, 107 6.5'W.

Figure 10 shows the original Park AOA at 64,67 Km (which showed low N/D) and (the?) two possible ways of unfolding the phases so that N/D stays low. The track to the East is the only one which is consistent with the radial velocity variation, so this must be the true track, and the AOA in the previously mentioned Table were unfolded according to this position. If Figure 10 and Figure 8 are compared, it can be seen that the selected track is in the right sector to immediately give low N/D without unfolding - which it did, but the AOA calculated from these measurements was wrong. Also, Figure 4 shows that the selected track is in the middle of a sidelobe of the combined antenna pattern - lucky!

Figure 11 shows the NASA and radar positions relative to Park.

VII Vz Accuracy

After a comment from C.H.Liu that our Vz values seemed too high, a test was done on the balloon data. Figure 12 shows a plot of radar Vz vs. time. The slope of this graph can be used to calculate the track speed. Let X be the distance along the track from the point at which the track is perpendicular to the radar line of sight, and R be the range to the track at this point, and V be the balloon speed along the track (Figure 13).

Then the radial component of velocity seen by the radar is

$$V_{rad} = V \sin(\theta), \quad (7)$$

which approximates (for small angles) to

Table 2. Radar data: original phases and Vrad; AOA for unfolded phases; N,E,height positions assuming nominal range; and signal strength (S/NdB). Phase calibrations used: 314,302,294 deg.

date/time UT	range Km	4V3 deg	4V2 deg	4V1 deg	Zen deg	Azim deg	N Km	E Km	height Km	Vrad m/s	S/N dB
4V3 and 4V2 unfolded											
88:241/2025	67	60-163	145	49.3	85.	4.8	50.6	43.7	*-4.22	8.2	
88:241/2030	64	55-165	109	47.9	81.	7.6	46.9	42.9	-2.47	13.1	
88:241/2035	64	63-150	88	47.3	77.	10.8	45.7	43.4	*-0.41	11.7	
88:241/2040	64	68-132	68	47.7	73.	14.2	45.1	43.1	0.92	12.0	
88:241/2045	64	72-122	53	47.8	70.	16.4	44.5	43.0	1.99	9.8	
88:241/2050	64	77-119	35	47.1	67.	18.3	43.2	43.6	2.74	7.3	
88:241/2050	67	93 -87	59	49.0	67.	19.9	46.5	44.0	3.81	3.5	
88:241/2055	67	79 -99	37	49.1	66.	20.8	46.2	43.9	3.88	8.0	
88:241/2100	67	69-101	30	50.1	66.	21.3	46.8	43.0	3.36	7.1	
88:241/2105	67	61-106	19	50.4	65.	21.9	46.8	42.7	*4.49	2.9	
88:241/2105	70	74 -79	43	52.2	65.	23.1	50.3	42.9	*2.17	3.6	
88:241/2110	70	65 -92	45	51.9	67.	21.7	50.6	43.2	2.24	5.8	
88:241/2115	70	56-102	38	51.8	67.	21.4	50.7	43.3	*1.49	5.6	
88:241/2120	73	42 -60	44	59.5	66.	25.9	57.3	37.1	2.30	4.3	
88:241/2125	73	44 -82	52	56.3	68.	22.9	56.3	40.5	*2.59	5.2	
88:241/2130	73	23 -63	47	62.4	67.	25.3	59.5	33.8	*1.72	4.9	
4V3 unfolded											
88:241/2010	73	35	127	137	47.1	91.	-1.0	53.5	49.7	*-1.81	6.2
88:241/2015	70	28	169	153	52.2	88.	1.6	55.3	42.9	*-5.73	4.7
88:241/2020	67	42	168	146	49.6	88.	2.0	51.0	43.4	*-5.52	11.0
88:241/2025	64	45	169	126	48.0	85.	3.8	47.4	42.8	-4.5	9.0

* means > 10% auto phase curvature

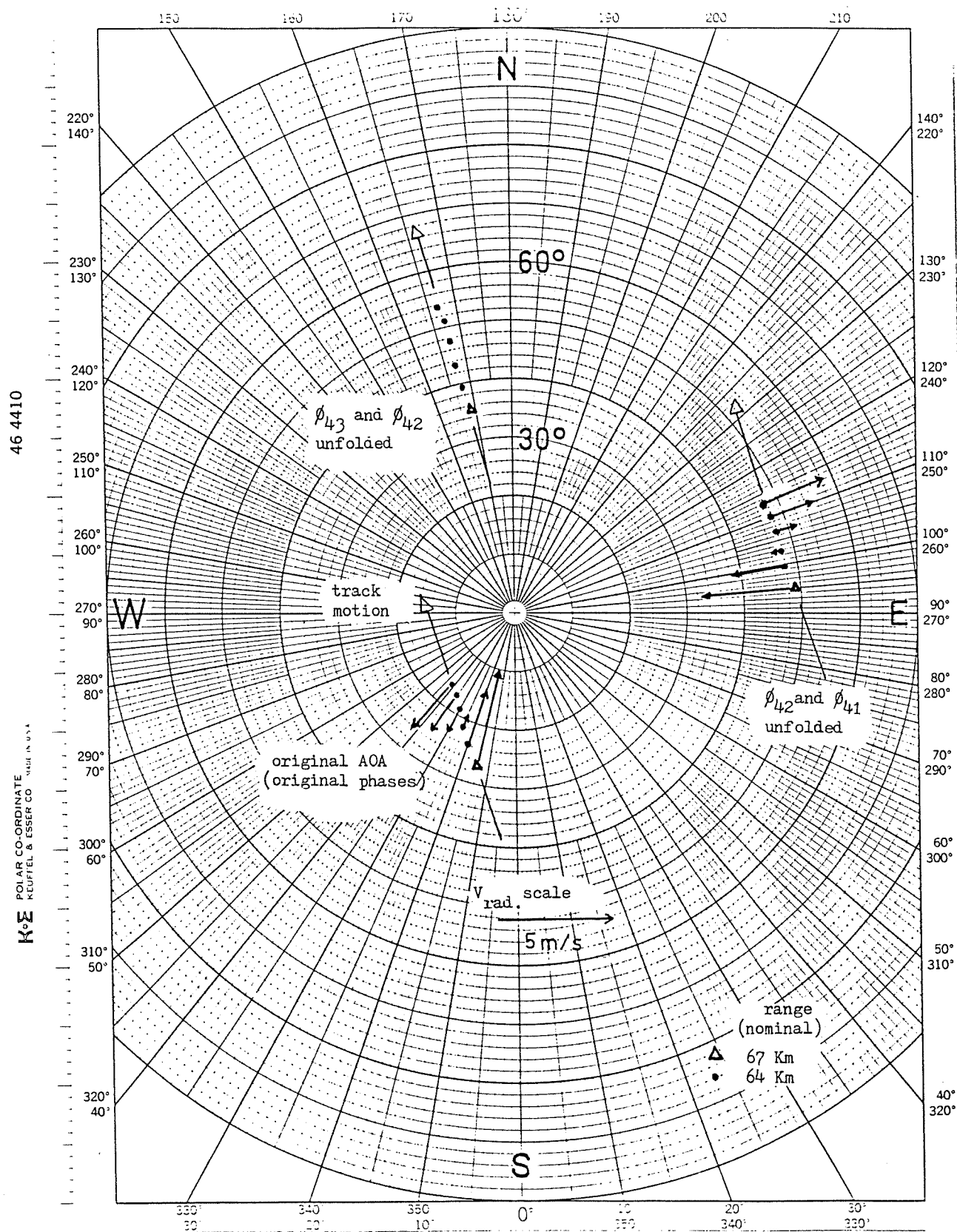


Figure 10.

FREE SPACE

POST NO BILLS

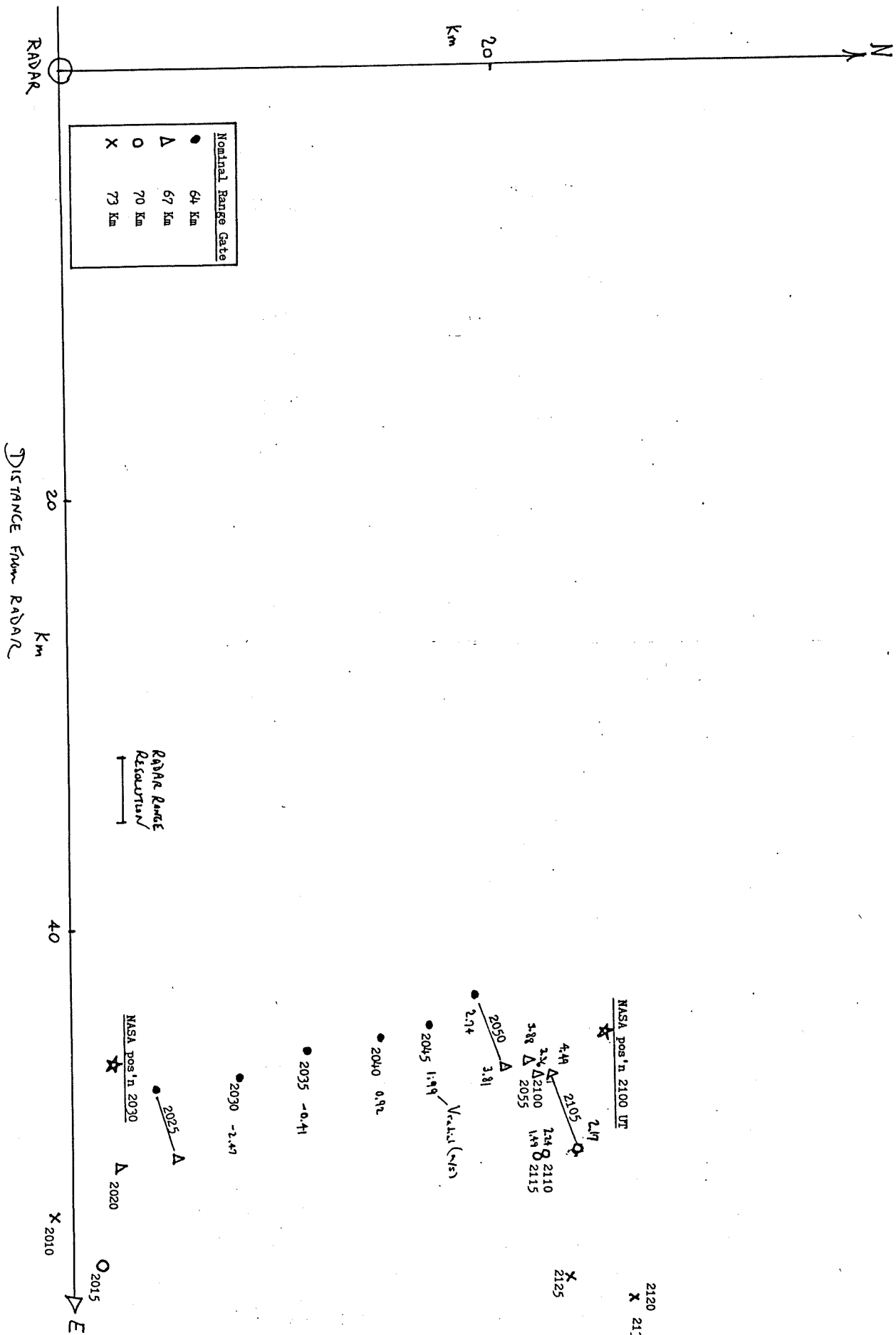


Figure 11.

Balloon Track: Signal and V_{radial} vs. time
(88:241/2010- UT)

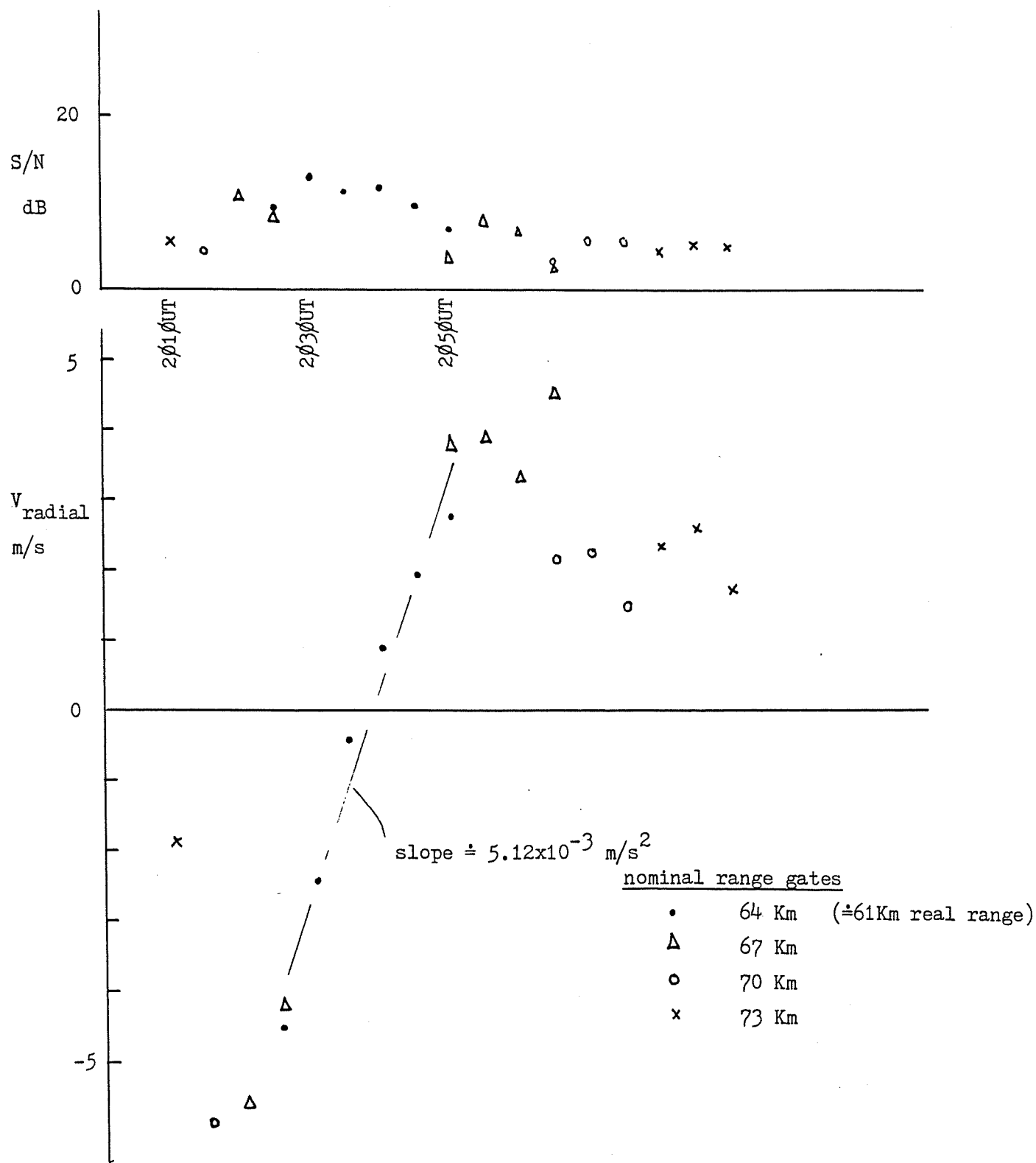


Figure 12.

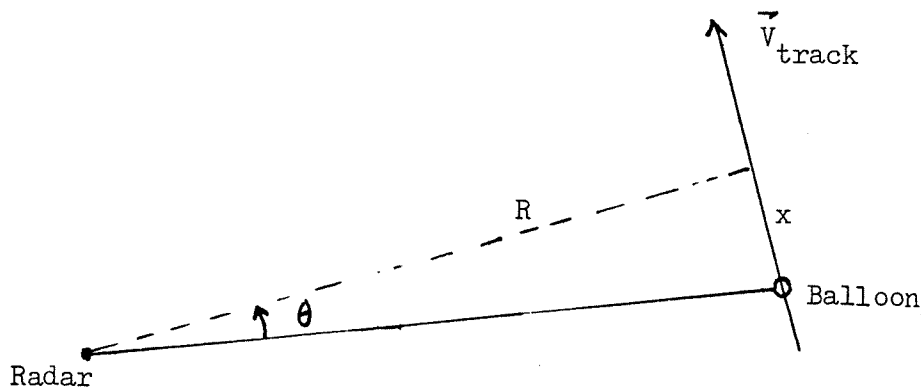


Figure 13.

$$V_{rad} = V \cdot x/R = V^2 t/R. \quad (8)$$

Thus the slope is V^2/R . Here R is $64-3 \text{ Km} = 61 \text{ Km}$ (as close as we can approximate), and the slope from Figure 12 is

$$5.12 \times 10^{-3} \text{ m/s}^2.$$

So the predicted track velocity is $V = 17.6 \text{ m/s}$. But the track velocity found from the changing AOA is only $V = 10.4 \text{ m/s}$. Therefore, because V_{rad} varies as the square of V , the measured V_{rad} must be to 2.8 times greater than that predicted from the motion along the track (a single point from the track was also checked, and confirms this ratio). Why is this?

VIII. Further V_z/V_{rad} discussion.

A test was carried out to see whether conversion to bit amplitudes could cause a bias, since no correction is made to the phase for these. If X_1, Y_1 are the I, Q components of a point in the bit-amplitude sequence, and X_2, Y_2 are another point at lag L from the first, then the auto correlation phase for lag L is

$$\phi(L) = \tan^{-1} \left\{ \frac{\overline{Y_1 \cdot X_2 - X_1 \cdot Y_2}}{\overline{X_1 \cdot X_2 + Y_1 \cdot Y_2 - X_1 \cdot X_2 - Y_1 \cdot Y_2}} \right\} \quad (9)$$

Because of noise from extreme digitization, each of the sums must be less than if there were no conversion to bit-amps. At a guess, each product should be reduced by the same absolute amount (viz. assume the conversion produces the same amount of noise in each X_1, Y_1 etc., and note that X_1, Y_1 etc. are statistically the same - being formed from rotating phasors). But normally the phase is very small (small Doppler) and so the numerator is much less than the denominator: thus the loss of "product" due to noise cancellation reduces the numerator

by a greater factor than the denominator; and so the measured phase is too small. For small angles by computer model, this reduction factor is about 1.6 (very similar to the effective bit-amplitude correction factor which would apply to small correlation magnitudes! , which is $1.5708 = \pi/2$ - probably not a coincidence, but haven't worked it out yet). If the phase goes above 45 deg, the numerator is greater than the denominator, and uncorrected Dopp_lers are too high.

The correction factor can be derived as follows: let $X_1 = \cos(2\pi t/T)$, $Y_1 = \sin(2\pi t/T)$, $X_2 = \cos(2\pi(t+l)/T)$, $Y_2 = \sin(2\pi(t+l)/T)$, where T is the Doppler period, and l is the lag, and refer to Figure 14.

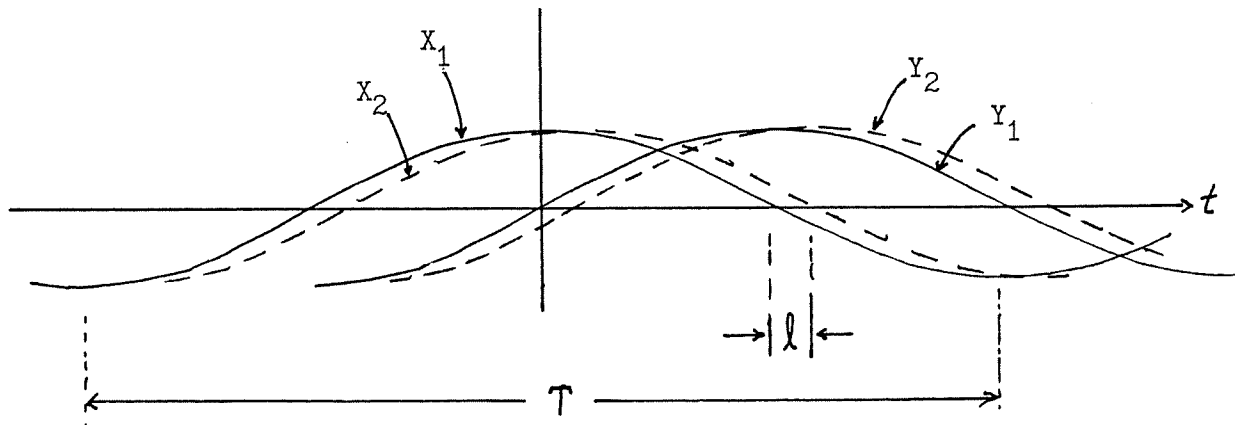


Figure 14.

Then for small lags, l , and bit-amps (1 if signal > zero, 0 if not), it can be seen that for example the probability

$$P(X_1 > 0 \text{ and } X_2 > 0) = \frac{T/2 - l}{T}$$

$$= P(Y_1 > 0 \text{ and } Y_2 > 0)$$

and

$$P(Y_1 > 0 \text{ and } X_2 > 0) = \frac{1}{4} + \frac{l}{T}$$

$$P(X_1 > 0 \text{ and } Y_2 > 0) = \frac{1}{4} - \frac{l}{T}$$

Also $\overline{X1} = \overline{X2} = \overline{Y1} = \overline{Y2} = 1/2$. Note that the mean values of (X1,X2) etc are just the probabilities above. Plugging these into equation 9 gives the phase, ϕ' , measured when bit-amps are used.

$$\tan(\phi') = \frac{2l}{T - 2l}$$

$$= \frac{\phi}{\frac{\pi}{2} - \phi} \quad (\text{for } 0 \leq \phi \leq \frac{\pi}{2})$$

where ϕ is the phase obtained with full amplitude correlations; which, for $l \ll T$ (small phases), reduces to

$$\phi' = \frac{4l}{T} \quad (\text{radians}).$$

If full amplitudes had been used, the phase would be simply $\phi = 2\pi l/T$.

Thus for small phases/Doppler frequencies, the bit-amplitude Doppler velocity is a factor of $\pi/2 = 1.5708$ below the real one. Note that these corrections are only important when the phase is small, as it is in Doppler measurements. The AOA phases have ~~u~~ similar errors when bit-amps are used, but since AOA is relatively insensitive to small errors (in our usage anyway AOA is found from large phases), these are not important.

Figure 15 shows corrections for V_z measurements and V_z phase "curvature" over the 1st 2 lags. If $P1^\circ$ and $P2^\circ$ are the phases at the 1st two lags, then V_z is the slope predicted at zero lag from:

$$V_z = (4P1 - P2) * 0.5 * FC \quad (\text{m/s}),$$

where $FC = (135/360)/(2*DT)$, where $DT = 0.5333$ sec, and radio wavelength = 135 m. The curvature CU is the difference in slope between lags 0-1 and 1-2, relative to slope 0-1, and is given by

$$CU = \text{ABS}(P2 - 2*P1)/2 * 100\%$$

Figure 16 shows errors for zero-lag cross phases. This plot is very close to the function $(\phi = 0 \rightarrow 360 \text{ deg.}) \quad 4.06 \sin(4\phi)$ in degrees. Note that this is the error in the uncalibrated phase (before subtracting the cable/Rx calibration value). If the maximum error occurred at the zenith, the error in measured zenith angle would be ~ 0.5 deg.

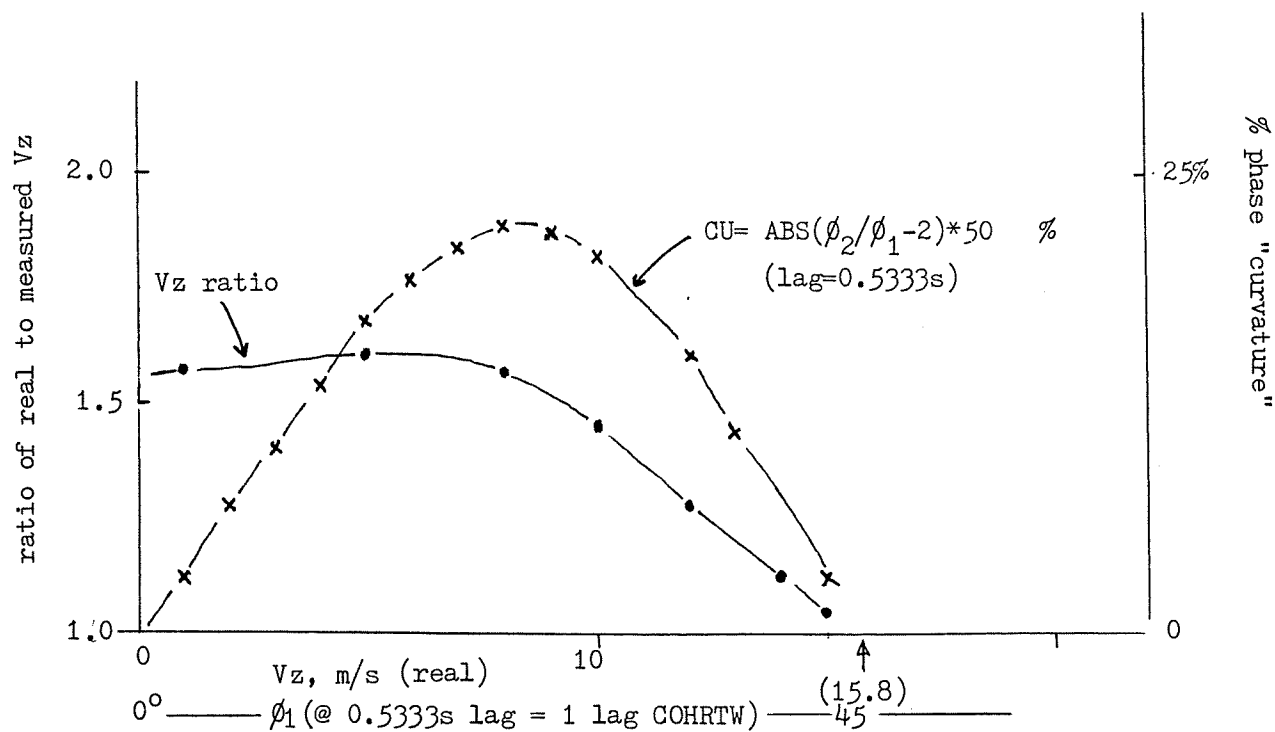


Figure 15.

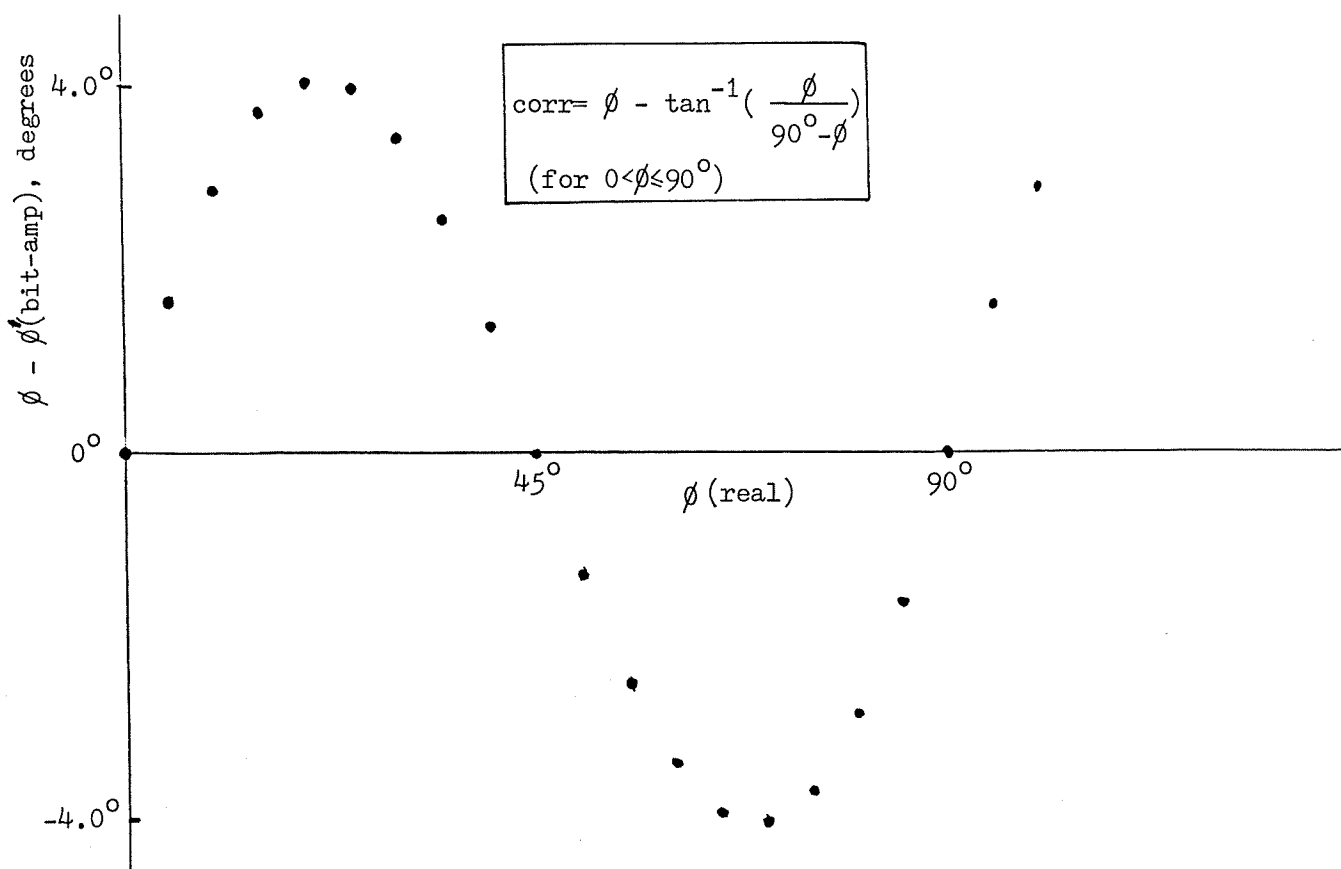


Figure 16. Correction which must be added to measured (bit-amp) phase to get real phase.

IX. Vz errors due to errors in I-Q quadrature and differences in channel gains.

These are not especially important for the COHRTW system, since 4 receivers are being averaged in the auto, but is added for completeness (and applies to the previous "Vz system" equipment).

Let $X1 = \cos(\omega.t)$, $Y1 = r.\sin(\omega.t+q)$, $X2 = \cos(\omega.t+\phi)$, $Y2 = r.\sin(\omega.t+q+\phi)$. So 'r' is the gain of the "Q" channel with respect to the "I"; and q is the error in quadrature, assumed small, and ω is the angular Doppler frequency. Plugging these into equation 9 gets the result:

$$\tan(\phi') = \frac{2.r.\cos(q)\sin(\phi)}{(1+r)\cos(\phi)}$$

(I have ignored the sign of the RHS, it may be -ve if equation 9 is followed exactly). Here ϕ' is the phase that would be measured on a mal-adjusted receiver, and ϕ is the real phase. Thus if $q \neq 0$ (quadrature error) or $r \neq 1$ (channel gain difference) the measured phase/Doppler velocity will be too small (based on a 1-point phase measurement).

X. Other sources of error

For very small Vz, there may be problems with "stationarity". An actual balloon radial velocity of 0.7 m/s would create 3 cycles of Doppler oscillation per record (4.5 min). This possibility was tested in a model, and found to cause no obvious bias. An additional "worry" is that such a low frequency will cause errors in the long term mean (LTM) which is used to convert directly to bit amplitudes. This is hard to test, but unlikely to cause error - in the present implementation, the LTM is a running average of 4096 integrated amplitudes at each height gate/gain, and the record length is 512. So there might be trouble if there were a single scatterer with less than a cycle of Doppler per record, but this would probably be due to non-stationarity, i.e. very distorted mean calculated from #1 matches.

So what's left: noise? - tests don't show any obvious bias with noise up to 20-50% amplitude of signal, and the noise is well below this in the balloon case.

If the reflector is on the outside of the balloon envelope, and the balloon is rotating slowly, the wrong Vz would be measured; presumably the rotation rate would have to be less than 1 turn per $\frac{1}{2}$ hour, or Figure 12 would not look so linear. For a "10 storey" balloon, say 30m diameter, one turn per hour would give a maximum Vz error of 0.05 m/s - so this doesn't look like a probable cause.

xi Addendum: Park log Jul.17'90

A piece of hardware was built to generate a 1.28 m/s Doppler component (sinusoidal output at 2.219 MHz). The resulting Vz was 0.85-0.95 m/s in normal C128 analysis at various heights. Applying the factor of 1.6 gives ~ 1.36 m/s: reasonably good agreement with the hardware design.

Balloon tracking: figure captions

Figure 1. Daytime signal strength vs. time (5 min resolution) for all heights. Each plot is signal amplitude normalized to the maximum in the day. Signal amplitude (in the COHRTW system) is defined as the RMS value of the mean square values of I and Q amplitudes found separately multiplied by the gain used at the particular height gate.

Figure 2. Scatter plots of heights of local maximum signal (with respect to the two adjacent height gates), 1 plot per day: 288 times compressed into 256 time bins. Resolution is 1 Km (found by parabolic fit to signal over the three height gates). Height scale may be incorrect by 1 or 2 Km. Year (8) and day number given. Plot starts at 0000 UT.

Figure 3. Park observatory antenna systems. The "Y" array is used for the COHRTW receiving system. The centre antenna is correlated with each of the outer ones. Spacing is 156 m.

Figure 4. Polar plot of combined Tx/Rx sensitivity pattern (dB), where Rx refers to one of the receiving antenna squares.

Figure 5. Phase calibration curves for 88:224-299, daytime only (0300-1355 UT excluded). No prior correction has been done for bit-amplitude errors. Since there is different signal "routing" in a receiver depending on selected gain, histograms have been separated according to gain. Individual phase differences with a related cross correlation magnitude < 0.2 are not used. The calibration value is the most probable value in a histogram.

Figure 6. There is no Figure 6. (5,6 were combined)

Figure 7. Spherical trigonometry for finding the angle difference between antenna pair vectors and the arriving ray (AOA). This is used to obtain equations for the phase differences, which are inverted so that AOA can be calculated from them.

Figure 8. A polar plot of real AOA, which is shaded to show where the measured phase differences (-180° to $+180^\circ$) will immediately give a normalized phase discrepancy of 1. In the centre area, where the NOD is zero, the correct AOA will be obtained from these phase differences. At the edges, the calculated AOA will not be correct - i.e. the real phase differences are beyond -180° to $+180^\circ$.

Figure 9. This shows the spherical trigonometry for a low centre antenna: i.e. all the pair vectors point above the horizon.

Figure 10. Shows the original track (which was in the proper location on Figure 8 to have zero NOD) and two possible tracks, still with "zero" NOD (neglecting minor measured phase fluctuations) found by "unfolding" one or more of the 3 phase differences: that is adding or subtracting 360° . Each resulting phase difference must be less than 416° for the "Y" array, because this is the maximum phase difference

possible given the array spacing of 156 m, with a RF wavelength of 135 m; so there are not as many possibilities as one might expect.

Figure 11. Shows the final radar determined balloon positions along with two positions from NASA. The measured Doppler velocity is written beside each point. UT (for the start of the 4.5 min record) is typed.

Figure 12. Shows signal/noise (approx. = signal, because little change in noise) and radial velocity (=COHRTW Vz) vs. time. Values are from Table 2.

Figure 13. Shows sketch for determining track velocity from slope of radial velocity vs. time shown in Figure 12.

Figure 14. Sketch for deriving bit-amplitude value of COHRTW Vz from real Vz. T is the Doppler period, ℓ is the auto-correlation lag.

Figure 15. Shows reduction factor in COHRTW Vz with respect to real Vz because of bit-amplitude usage. Curvature is also shown, because these values are flagged in the COHRTW system (see Table 2). Values beyond 15 m/s have not been calculated, but it is probably possible to work the Vz factor out from the next Figure.

Figure 16. Shows the correction to be applied to raw (before "calibrating") phases. The "sinusoid" continues out to 360° and can be approximated pretty accurately by $4.06\sin(4\phi)$.

Appendix A :Effect of lower Centre antenna

<pre> 10-MAY-1989 15:34120 1 ED ANTFLD1.FOR 1 C... INCLUDE CENTRE ANT 25FT BELOW OUTER(20.4 DEG) 11:99999 1 C... INCLUDE CENTRE ANT 25FT BELOW OUTER(20.4 DEG) 2 C.... CHECK ANTENNA ZEN/AZ FOLDING FOR 'Y' RX ARRAY 3 C.... PROGG ANTFLD.FOR 4 DIMENSION RXP(2,3),PHI(3),PHIA(3) 5 DIMENSION ZN(72),AZIMUT(72) 6 CHARACTER*1 HS(72),GOOD,BAD 7 DATA GOOD,BAD/'1','0'/ 8 DATA RXP/156.,90.,156.,210.,156.,330.,/XX,BLANK/XX',OK'/ 9 PP=3,14159/180. 10 PIZ=6.28318 11 ISKIP=1 12 DO 400 IZ=6,75,5 13 DO 30 K=1,72 14 HS(K)=BAD 15 ZN(K)=0 16 AZIMUT(K)=0 17 30 CONTINUE 18 ZEN=(IZ-1)*0.01 19 DO 100 IAZ=1,18 20 AZ=(IAZ-1)*20-0.01 21 X=SIN(ZEN*PP)*COS(AZ*PP) 22 Y=SIN(ZEN*PP)*SIN(AZ*PP) 23 CA=COS(ZEN*PP) 24 DPR=SQRT(7.64**2+156.**2) 25 SE=7.34/DPR 26 CE=156./DPR 27 C... GET PHASE DIFFS 28 DO 20 L=1,3 29 CP=COS(RXP(2,L)*PP) 30 SP=SIN(RXP(2,L)*PP) 31 PHI(L)=-DPR/135.*360.*(SE*CA+CE*CP*X+CE*SP*Y) 32 PH=PHI(L)+20.4 33 20 PHIA(L)=PH 34 X1=PHIA(1) 35 X2=PHIA(2) 36 X3=PHIA(3) 37 HS(IAZ)=BAD 38 IF(ABS(X1+X2+X3)/(ABS(X1)+ABS(X2)+ABS(X3)).LT.0.3)HS(IAZ)=GOOD 39 CALL ARRIVE(X1,X2,X3,Z,A,MESS) 40 IF(MESS.EQ.1)GO TO 100 41 IF(A.LT.0.)A=A+360 42 ZN(IAZ)=Z 43 AZIMUT(IAZ)=A 44 IF(ISKIP.EQ.1)WRITE(3,1221) 45 ISKIP=0 46 1221 FORMAT(1X,/,/,/) 47 C....WRITE(3,1091)ZEN,AZ,PHI,PHIA,AA,X1,X2,X3,Z,A 48 1091 FORMAT(1X,2F6.0,1X,3F6.0,2X,3F6.0,1X,A2,1X,3F6.0, 49 1F6.1,F6.0) 50 100 CONTINUE 51 WRITE(4,1222)ZEN,(ZN(J),J=1,18) 52 WRITE(21,1222)ZEN,(AZIMUT(J),J=1,18) 53 1222 FORMAT(1X,'ZEN=',F4.0,' ',18F6.1) 54 WRITE(3,1022)ZEN,(HS(L),ZN(L),AZIMUT(L),L=1,18) 55 1022 FORMAT(/,1X,F5.1,' ',12(A1,F4.1,F5.0);5(/,7X,12(A1, 56 1F4.1,F5.0))) 57 400 CONTINUE 58 STOP 59 END </pre>	<pre> 60 SUBROUTINE ARRIVE(PHI1,PHI2,PHI3,ZENITH,AZIN,MESS) 61 DIMENSION XPH(3) 62 DATA PP/1.745329E-2/,ISKIP/1/ 63 C--- 'Y' ARRAY 64 DATA D43,D42,D41,PSI14,PSI24,PSI34/156.,156.,156., 65 190.,210.,330./ 66 C.... D14=ANT SPACING,PSI14= DIR 4=>1 ETC 67 C.....'Y' ARRAY: #1=NORTH,#2=SOUTH,#3=EAST,#4=CENTRE 68 IF(ISKIP.EQ.0)GO TO 5 69 ISKIP=0 70 C14=COS(PSI14*PP) 71 C24=COS(PSI24*PP) 72 C34=COS(PSI34*PP) 73 S14=SIN(PSI14*PP) 74 S24=SIN(PSI24*PP) 75 S34=SIN(PSI34*PP) 76 C...ASSUME EQUILATERAL 77 DD=135/D43/360. 78 A11=C14**2+C24**2+C34**2 79 A22=S14**2+S24**2+S34**2 80 A12=C14*S14+C24*S24+C34*S34 81 AA=A11*A22-A12**2 82 S=0 83 S1=0 84 XPH(1)=PHI1 85 XPH(2)=PHI2 86 XPH(3)=PHI3 87 MESS=0 88 DO 20 K=1,3 89 S=S+XPH(K) 90 S1=S1+ABS(XPH(K)) 91 20 CONTINUE 92 ANPD=ABS(S)/S1 93 IF(ANPD.LE.0.3)GO TO 30 94 MESS=1 95 RETURN 96 30 B1=(C14*XPH(1)+C24*XPH(2)+C34*XPH(3))*DD 97 B2=(S14*XPH(1)+S24*XPH(2)+S34*XPH(3))*DD 98 A=(B1*A22-B2*A12)/AA 99 B=(A11*B2-A12*B1)/AA 100 PD14=(A*C14+B*S14)/DD 101 PD24=(A*C24+B*S24)/DD 102 C.... AVG ZEN AND AZ OF SCATTER 103 Q14=PD14/360*135./D43 104 Q24=PD24/360.*135./D42 105 AZIN=0 106 IF(Q24*S14-Q14*S24.EQ.0.)GO TO 50 107 AZIN=ATAN2(Q14*C24-Q24*C14,Q24*S14-Q14*S24) 108 50 Q14=Q14/(COS(AZIN)*C14+SIN(AZIN)*S14) 109 IF(ABS(Q14).LT.1.)GO TO 45 110 ZENITH=0 111 TYPE1336 112 1336 FORMAT(1X,'BAD ARG FOR ASIN,ZEN SET=0.:') 113 MESS=1 114 GO TO 48 115 C....DONT KNOW WHY ZEN COMES OUT -VE,BUT REV SGN HERE FOR NOW 116 45 ZENITH=-ASIN(Q14) 117 48 AZIN=AZIN/PP 118 ZENITH=ZENITH/PP 119 RETURN 120 END </pre>
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Appendix A cont'd

\$ EXE ANTFLDI \$FORTRAN ANTFLDI \$LINK ANTFLDI \$DEFI/USER_MODE SYS\$INPUT SYS\$COMMAND \$RUN ANTFLDI																			
		TRUE AZIMUTH →																	
FORTRAN STOP		0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340
\$ TY FOR001.DAT		0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340
ZEN= 5.1		5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
ZEN= 10.1		10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
ZEN= 15.1		15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0	15.0
ZEN= 20.1		20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
ZEN= 25.1		25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
ZEN= 30.1		30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0
ZEN= 35.1		35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0
ZEN= 40.1		40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0	40.0
ZEN= 45.1		45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0
ZEN= 50.1		50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
ZEN= 55.1		55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0	55.0
ZEN= 60.1		60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
ZEN= 65.1		65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0	65.0
ZEN= 70.1		70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
		CALC. AZIMUTH VS. TRUE ZEN & AZIM																	
		0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340
\$ TY FOR021.DAT		0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340
ZEN= 5.1																			

Appendix B

Transmitter antenna + 1 Rx antenna Pattern Calculation

```

1199999 TXPATN.FOR
1 C.... RE ENTERED FROM OLD PRINTOUT AND MODIF.
2 C.... PARK TX PATTERN, FIND PHASE OF SIDELobe
3 DIMENSION DM(50,50)
4 COMMON/PLTPRN/ISTEP,HEAD,ITYP
5 CHARACTER*76 HEAD
6 DATA DM/2500*1.E8/
7 P=180./3.141593
8 P90=3.14159265358979/2.
9 IX=37
10 IY=37
11 DX=0.8
12 DY=0.8
13 TYPE1566
14 1566 FORMAT(IX,'WHAT CONTOUR STEP(DB)?')
15 ACCEPT*,ISTEP
16 CALL PWRDB(0.,0.,DB)
17 DO 200 KX=1,37
18 TYPE 222,KX
19 222 FORMAT(IX,'KX=',I3)
20 DO 200 KY=1,37
21 C***** FIND AZIM & ZEN
22 XX=KX-19
23 YY=KY-19
24 R=SQRT(XX*XX+YY*YY)
25 ZEN=R*5/P
26 IF(ZEN,GT,P90)GOTO180
27 IF(KY-19,LE,0)GOTO 20
28 AZIM=P90
29 IF(KX-19,GE,0)AZIM=P90
30 GOTO 30
31 20 AZIM=ATAN2(KX-19.,KY-19.)
32 30 CALL PWRDB(ZEN,AZIM,DB)
33 DM(KX,KY)=DB
34 180 CONTINUE
35 200 CONTINUE
36 C***** NORMAL WRT ZEN=0
37 AMAX=DM(19,19)
38 DO 220 KX=1,IX
39 DO 220 KY=1,IY
40 IF(DM(KX,KY).LT.1.E4)DM(KX,KY)=AMAX-DM(KX,KY)
41 220 CONTINUE
42 TYPE 1111,AMAX
43 1111 FORMAT(IX,'MAX POWER=',F6.1,'DB')
44 TYPE1522
45 1522 FORMAT(IX,' WHAT HEADER?')
46 ACCEPT 1112,HEAD
47 1112 FORMAT(A76)
48 OPEN(23,FILE='PLOT.DAT',STATUS='NEW')
49 CALL CNTPLT(DM,DX,DY,IX,IY,0)
50 STOP
51 END

```

```

52 SUBROUTINE PWRDB(ZEN,AZIM,DB)
53 P=180./3.1415926
54 30 C=COS(ZEN)
55 A=SIN(ZEN)*COS(AZIM)
56 B=SIN(ZEN)*SIN(AZIM)
57 C...DIPole FACTOR(AMP), USE A FOR N-S DIPOLES; B FOR E-W
58 DF=SQRT(1-A*A)
59 C...END OF DIPOLE FACTOR
60 SUMC=0
61 SUMS=0
62 DO 100 I=1,4
63 C.... PP=0;IF(I,LT,3)PP=3.14159
64 C...A BOVE USED TO BE INT TO GET ANT OUT OF PHASE
65 X=(I-2.5)*0.5
66 DO 100 J=1,4
67 Y=(J-2.5)*0.5
68 DO 100 K=1,2
69 Z=(K-1.5)*0.5
70 D=(X*A+Y*B+Z*C)/SIGN(1.,C)*6.28318
71 DD=D/6.28318
72 L=(I-1)*8+(J-1)*2+K
73 C.... COS FOR SINGLE DIPOLE
74 SUMC=SUMC+COS(D+3.14159*(K-1))*DF
75 SUMS=SUMS+SIN(D+3.14159*(K-1))*DF
76 100 CONTINUE
77 SUM=SQRT(SUMC**2+SUMS**2)
78 IF(ZEN,LT,0.0001)SUM1=SUM
79 GAIN=20.*ALOG10(ABS(SUM/SUM1))
80 PB=ABS(SUM/SUM1)**2
81 PHI=ATAN2(SUMS,SUMC)*P
82 C....X=N,Y=E, N-S DIPOLES USED, RX CONTRIB TO GAIN
83 SUMC=0
84 SUMS=0
85 X=0
86 DO 150 J=1,2
87 Y=(J-1.5)*0.5
88 DO 150 K=1,2
89 Z=(K-1.5)*0.5
90 D=(A*X+B*Y+C*Z)/SIGN(1.,C)*6.28318
91 SUMC=SUMC+COS(D+3.14159*(K-1))*DF
92 SUMS=SUMS+SIN(D+3.14159*(K-1))*DF
93 150 CONTINUE
94 SUMRX=SQRT(SUMC**2+SUMS**2)
95 IF(ZEN,LT,0.0001)SUM1RX=SUMRX
96 GRX=20.*ALOG10(SUMRX/SUM1RX)
97 DB=GRX+GAIN
98 RETURN
99 END

```

Angle of Arrival Calculation from Zero Lag Phases.

```

67      VZ(K1)=99
68      MSVZ(K1)=0
69      VNE(K1,1)=-99
70      VNE(K1,2)=-99
71      IH=IREC(1,K)
72      IF(IH.NE.K-1)GOTO100
73      IF(IGAIN(K).GT.7)GOTO100
74      IVZ=IREC(8,K)*256+IREC(9,K)
75      IF(IVZ.EQ.0)GOTO45
76      VZ(K1)=(IVZ-T0)/100.
77      MSVZ(K1)=IREC(10,K)/64
78      AN2=0
79      AN1=0
80      IVTR=IREC(14,K)*256+IREC(15,K)
81      IF(IVTR.EQ.0)GOTO45
82      PTR=IREC(16,K)*256+IREC(17,K)
83      VNE(K1,1)=IVTR/10.*COS(PTR*PP)
84      VNE(K1,2)=IVTR/10.*SIN(PTR*PP)
85      45 DO 50 L=1,3
86      L1=(L-1)*2+2
87      LV=IREC(L1,K)+IREC(L1+1,K)*256
88      IFAZ=MOD(LV,512)
89      C.... IFAZ ALWAYS 0-360
90      C.... USE IFAZ FOR PHASE DIFF HISTOGRAMS
91      IPHI(L)=IFAZ-IPCOR(L)
92      IF(IPHI(L).GT.180)IPHI(L)=IPHI(L)-360
93      IF(IPHI(L).LT.-180)IPHI(L)=IPHI(L)+360
94      AN1=AN1+IPHI(L)
95      AN2=AN2+IABS(IPHI(L))
96      50 CONTINUE
97      AW(K1,2)=ABS(AN1)/AN2*100+400
98      C.... IF(AN2.EQ.0..OR..ABS(AN1)/AN2.GT.0.3)GOTO100
99      PHI1=IPHI(1)+IUNF(1)*360
100     PHI2=IPHI(2)+IUNF(2)*360
101     PHI3=IPHI(3)+IUNF(3)*360
102     CALL ARRIVE(PHI1,PHI2,PHI3,ZEN,AZIM,MESS)
103     IF(MESS.EQ.1)GOTO100
104     X=IKM*SIN(ZEN*PP)*COS(AZIM*PP)
105     Y=IKM*SIN(ZEN*PP)*SIN(AZIM*PP)
106     IF(IXK.LE.73)WRITE(23,1211)IYR,IDY,IHR,IMIN,IKM,IPHI,ZEN,AZIM,
107     IX,Y
108     1211 FORMAT(1X,I2,' ',I3,'/',I2,' ',I4,'KM',1X,3I4,F6.1,F6.0,2F6.1)
109     AW(K1,1)=ZEN
110     AW(K1,2)=AZIM
111     100 CONTINUE
112     IF(KTYP.EQ.1)GOTO120
113     IF(KTYP.EQ.0)GOTO110
114     IF(KTYP.EQ.2)GOTO105
115     WRITE(3,1471)IYR,IDY,IHR,IMIN,((VNE(L,M),M=1,2),L=1,8)
116     1471 FORMAT(1X,I2,' ',I3,'/',I2,' ',I4,'KM',1X,3I4,F6.1,F6.0,2F6.1)
117     GO TO 10
118     105 WRITE(3,1472)IYR,IDY,IHR,IMIN,((SIGNAL(L,M),M=1,2),L=1,8)
119     1472 FORMAT(1X,I2,' ',I3,'/',I2,' ',I4,'dB',1X,3I4,F6.1,F6.0,2F6.1)
120     GO TO 10
121     110 WRITE(3,1478)IYR,IDY,IHR,IMIN,(MSVZ(L),VZ(L),L=1,8)
122     1478 FORMAT(1X,I2,' ',I3,'/',I2,' ',I4,'dB',1X,3I4,F6.1,F6.0,2F6.1)
123     GOTO 10
124     120 WRITE(3,1477)IYR,IDY,IHR,IMIN,((AW(L,M),M=1,2),L=1,8)
125     1477 FORMAT(1X,I2,' ',I3,'/',I2,' ',I4,'dB',1X,3I4,F6.1,F6.0,2F6.1)
126     GO TO 10
127     800 type1455
128     1455 format(1X,'read error')
129     go to 10
130     990 STOP
131     END

```

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```

132 SUBROUTINE ARRIVE(PHI1,PHI2,PHI3,ZENITH,AZIM,MESS)
133 DIMENSION XPH(3)
134 C--- "Y" ARRAY
135 DATA PP/1.745329E-2/,ISKIP/1/
136 C.....LOOP ARRAY
137 DATA D43,D42,D41,PSI14,PSI24,PSI34/156.,156.,156.,
138 190.,210.,330./
139 C.... D14=ANT SPACING,PSI14= DIR 4-2 ETC
140 C.... #1=NORTH,#2=SOUTH,#3=WEST,#4=CENTRE!!! LOOP ARRAY !! ONLY !!
141 IF (ISKIP,EQ,0)GOTO5
142 ISKIP=0
143 C14=COS(PSI14*PP)
144 C24=COS(PSI24*PP)
145 C34=COS(PSI34*PP)
146 S14=SIN(PSI14*PP)
147 S24=SIN(PSI24*PP)
148 S34=SIN(PSI34*PP)
149 C...ASSUME EQUILATERAL
150 DD=135/D43/360.
151 A11=C14*#2+C24*#2+C34*#2
152 A22=S14*#2+S24*#2+S34*#2
153 A12=C14*S14+C24*S24+C34*S34
154 AA=A11*A22-A12*#2
155 5 S=0
156 S1=0
157 XPH(1)=PHI1
158 XPH(2)=PHI2
159 XPH(3)=PHI3
160 MESS=0
161 DO 20 K=1,3
162 S=S+XPH(K)
163 S1=S1+ABS(XPH(K))
164 20 CONTINUE
165 AMPD=ABS(S)/S1
166 IF (AMPD,LE,0.3)GOTO 30
167 MESS=1
168 RETURN
169 30 B1=(C14*XPH(1)+C24*XPH(2)+C34*XPH(3))*DD
170 B2=(S14*XPH(1)+S24*XPH(2)+S34*XPH(3))*DD
171 A=(B1*A22-B2*A12)/AA
172 B=(A11*B2-A12*B1)/AA
173 PD14=(A*C14+B*S14)/DD
174 PD24=(A*C24+B*S24)/DD
175 C.... AVG ZEN AND AZ OF SCATTER
176 Q14=PD14/360*135./D43
177 Q24=PD24/360.*135./D42
178 AZIM=0
179 IF (Q24*S14-Q14*S24,EQ,0.)GOTO 50
180 AZIM=ATAN2(Q14*C24-Q24*C14,Q24*S14-Q14*S24)
181 50 Q14=Q14/(COS(AZIM)*C14+SIN(AZIM)*S14)
182 IF (ABS(Q14),LT,1.)GOTO45
183 ZENITH=0
184 TYPE1336
185 1336 FORMAT(1X,'BAD ARG FOR ASIN,ZEN SET=0..')
186 MESS=1
187 GO TO 48
188 C....DONT KNOW WHY ZEN COMES OUT -VE,BUT REV SGN HERE FOR NOW
189 45 ZENITH=-ASIN(Q14)
190 48 AZIM=AZIM/PP
191 ZENITH=ZENITH/PP
192 RETURN
193 END
*DUIT

```

